Chapter 7
Vapor-Power Cycle

4.1 Introduction
Steam power plants are the major sources of power generation. They operate essentially on the principle that steam generated in the boiler passes through steam turbine and the expansion process of steam in the steam turbine results in work output. The steam exhausted from the turbine is passed through the steam condenser or process heaters. The condensate collected is then supplied to the boiler at an elevated boiler pressure by means of a feed-pump, completing the vapor-power cycle.

Figure 4.1 Components of a simple vapor power plant.

4.2 The Carnot Vapor Cycle
The Carnot cycle is the most efficient cycle operating between two specified temperature limits. Thus it is natural to look at the Carnot cycle first as a prospective ideal cycle for vapor power plants. If we could, we would certainly adopt it as the ideal cycle. As explained below, however, the Carnot cycle is not a suitable model for power cycles. Throughout the discussions, we assume
steam to be the working fluid since it is the working fluid predominantly used in vapor power cycles.

Consider a steady-flow Carnot cycle executed within the saturation dome of a pure substance, as shown in Fig. 4.2 Heat is supplied to the working fluid at constant temperature in a boiler (process 1-2), the fluid expanded isentropically in a turbine (process 2-3), then condensed at constant temperature in the condenser (process 3-4), and compressed isentropically by a compressor to the initial state (process 4-1).

![T-s diagram of Carnot vapor cycle.](image)

**Figure 4.2 T-s diagram of Carnot vapor cycle.**

The thermal efficiency of the Carnot cycle, $\eta_{\text{Carnot}}$, which is the highest for any cycle operating between the source temperature $T_1$, and the sink temperature $T_3$ is given by:

$$\eta_{\text{Carnot}} = 1 - \frac{T_3}{T_1} \quad (4.1)$$

**Limitations of Carnot Cycle**

Though Carnot cycle is simple (thermodynamically) and has the highest thermal efficiency for given values of $T_1$ and $T_2$, yet it is extremely difficult to operate in practice because of the following reasons:

- Limiting the heat transfer processes to two-phase systems, however, severely limits the maximum temperature that can be used in the cycle (it has to remain under the critical-
point value, which is 374°C for water). Limiting the maximum temperature in the cycle also limits the thermal efficiency. Any attempt to raise the maximum temperature in the cycle involves heat transfer to the working fluid in a single phase, which is not easy to accomplish isothermally.

- The isentropic expansion process (process 2-3) can be approximated closely by a well-designed turbine. However, the quality of the steam decreases during this process, as shown on the T-s diagram in Fig. 4.2. Thus the turbine has to handle steam with low quality, that is, steam with a high moisture content. The impingement of liquid droplets on the turbine blades causes erosion and is a major source of wear. Thus steam with qualities less than about 90 percent cannot be tolerated in the operation of power plants.

- The isentropic compression process (process 4-1) involves the compression of a liquid–vapor mixture to a saturated liquid. There are two difficulties associated with this process. First, it is not easy to control the condensation process so precisely as to end up with the desired quality at state 4. Second, it is not practical to design a compressor that handles two phases.

- The cycle is still more difficult to operate in practice with superheated steam due to the necessity of supplying the superheat at constant temperature instead of constant pressure.

Therefore, the Carnot cycle cannot be realized in practice, but sets the upper limit to which the cycle efficiency of any thermal plan can be raised

### 4.3 Rankine Cycle

Many of the impracticalities associated with the Carnot cycle can be eliminated by superheating the steam in the boiler and condensing it completely in the condenser, as shown schematically on a T-s diagram in Figure 4.3. The cycle that results is the **Rankine cycle**, which is the ideal cycle for vapor power plants. The ideal Rankine cycle does not involve any internal irreversibilities and consists of the following four processes:

1-2 Isentropic compression in a pump
2-3 Constant pressure heat addition in a boiler
3-4 Isentropic expansion in a turbine
4-1 Constant pressure heat rejection in a condenser
Figure 4.3 The simple ideal Rankine cycle

Water enters the pump at state 1 as saturated liquid and is compressed isentropically to the operating pressure of the boiler. The water temperature increases somewhat during this isentropic compression process due to a slight decrease in the specific volume of water. The vertical distance between states 1 and 2 on the T-s diagram is greatly exaggerated for clarity.

Water enters the boiler as a compressed liquid at state 2 and leaves as a superheated vapor at state 3. The boiler is basically a large heat exchanger where the heat originating from combustion gases, nuclear reactors, or other sources is transferred to the water essentially at constant pressure. The boiler, together with the section where the steam is superheated (the superheater), is often called the steam generator.

The superheated vapor at state 3 enters the turbine, where it expands isentropically and produces work by rotating the shaft connected to an electric generator. The pressure and the temperature of steam drop during this process to the values at state 4, where steam enters the condenser. At this state, steam is usually a saturated liquid–vapor mixture with a high quality.

Steam is condensed at constant pressure in the condenser, which is basically a large heat exchanger, by rejecting heat to a cooling medium such as a lake, a river, or the atmosphere. Steam leaves the condenser as saturated liquid and enters the pump, completing the cycle. In areas where water is precious, the power plants are cooled by air instead of water. This method of cooling, which is also used in car engines, is called dry cooling.
Remembering that the area under the process curve on a $T$-$s$ diagram represents the heat transfer for internally reversible processes, we see that the area under process curve 2-3 represents the heat transferred to the water in the boiler and the area under the process curve 4-1 represents the heat rejected in the condenser. The difference between these two (the area enclosed by the cycle curve) is the net work produced during the cycle.

**Energy Analysis of the Ideal Rankine Cycle**

All four components associated with the Rankine cycle (the pump, boiler, turbine, and condenser) are steady-flow devices, and thus all four processes that make up the Rankine cycle can be analyzed as steady-flow processes. The kinetic and potential energy changes of the steam are usually small relative to the work and heat transfer terms and are therefore usually neglected. Then the steady-flow energy equation per unit mass of steam reduces to

$$\left(q_{in} - q_{out}\right) + \left(w_{in} - w_{out}\right) = h_e - h_i \quad (kJ/kg) \quad (4.2)$$

The boiler and the condenser do not involve any work, and the pump and the turbine are assumed to be isentropic. Then the conservation of energy relation for each device can be expressed as follows:

**Pump** ($q=0$)  \hspace{1cm} $w_{pump, in} = h_2 - h_i$ \hspace{1cm} Or \hspace{1cm} $w_{pump, in} = v(P_2 - P_1)$ \hspace{1cm} (4.3)

Where  \hspace{1cm} $h_i = h_f @ P_1$ \hspace{1cm} and \hspace{1cm} $v = v_f @ P_1$

**Boiler** ($w=0$)  \hspace{1cm} $q_{in} = h_3 - h_2$ \hspace{1cm} (4.4)

**Turbine** ($q=0$)  \hspace{1cm} $w_{turb, out} = h_3 - h_4$ \hspace{1cm} (4.5)

**Condenser** ($w=0$)  \hspace{1cm} $q_{out} = h_4 - h_i$ \hspace{1cm} (4.6)

The **thermal efficiency** of the Rankine cycle is determined from

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} \quad (4.7)$$

Where  \hspace{1cm} $w_{net} = q_{in} - q_{out} = w_{turb, out} - w_{pump, in}$ \hspace{1cm} (4.8)
Deviation of Actual Vapor Power Cycles from Idealized Ones

The actual vapor power cycle differs from the ideal Rankine cycle, as illustrated in Figure 4.4, as a result of irreversibilities in various components. Fluid friction and heat loss to the surroundings are the two common sources of irreversibilities.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a somewhat lower pressure. Also, the pressure at the turbine inlet is somewhat lower than that at the boiler exit due to the pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate for these pressure drops, the water must be pumped to a sufficiently higher pressure than the ideal cycle calls for. This requires a larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.
Of particular importance are the irreversibilities occurring within the pump and the turbine. A pump requires a greater work input, and a turbine produces a smaller work output as a result of irreversibilities. Under ideal conditions, the flow through these devices is isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

\[ \eta_p = \frac{w_p}{w_a} = \frac{h_{2s} - h_i}{h_{2a} - h_i} \]  \hspace{1cm} (4.9)

And

\[ \eta_T = \frac{w_T}{w_s} = \frac{h_{4s} - h_{4a}}{h_{3s} - h_{4s}} \]  \hspace{1cm} (4.10)

where states 2a and 4a are the actual exit states of the pump and the turbine, respectively, and 2s and 4s are the corresponding states for the isentropic case (Figure 4.4 b).

Other factors also need to be considered in the analysis of actual vapor power cycles. In actual condensers, for example, the liquid is usually subcooled to prevent the onset of cavitation, the rapid vaporization and condensation of the fluid at the low-pressure side of the pump impeller, which may damage it. Additional losses occur at the bearings between the moving parts as a result of friction. Steam that leaks out during the cycle and air that leaks into the condenser represent two other sources of loss. Finally, the power consumed by the auxiliary equipment such as fans that supply air to the furnace should also be considered in evaluating the overall performance of power plants.

The effect of irreversibilities on the thermal efficiency of a steam power cycle is illustrated below with an example.

4.4 Increase the Efficiency of the Rankine Cycle

Steam power plants are responsible for the production of most electric power in the world, and even small increases in thermal efficiency can mean large savings from the fuel requirements. Therefore, every effort is made to improve the efficiency of the cycle on which steam power plants operate.

The basic idea behind all the modifications to increase the thermal efficiency of a power cycle is the same: *Increase the average temperature at which heat is transferred to the working fluid in the*
boiler, or decrease the average temperature at which heat is rejected from the working fluid in the condenser. That is, the average fluid temperature should be as high as possible during heat addition and as low as possible during heat rejection. Next we discuss three ways of accomplishing this for the simple ideal Rankine cycle.

**Lowering the Condenser Pressure (Lowers \( T_{\text{low,avg}} \))**

Steam exists as a saturated mixture in the condenser at the saturation temperature corresponding to the pressure inside the condenser. Therefore, lowering the operating pressure of the condenser automatically lowers the temperature of the steam, and thus the temperature at which heat is rejected.

The effect of lowering the condenser pressure on the Rankine cycle efficiency is illustrated on a \( T-s \) diagram in Figure (4.6). For comparison purposes, the turbine inlet state is maintained the same. The colored area on this diagram represents the increase in net work output as a result of lowering the condenser pressure from \( P_4 \) to \( P'_4 \). The heat input requirements also increase (represented by the area under curve \( 2'-2 \)), but this increase is very small. Thus the overall effect of lowering the condenser pressure is an increase in the thermal efficiency of the cycle.

![Figure 4.6 The effect of lowering the condenser pressure on the ideal Rankine cycle.](image)

To take advantage of the increased efficiencies at low pressures, the condensers of steam power plants usually operate well below the atmospheric pressure. This does not present a major problem since the vapor power cycles operate in a closed loop. However, there is a lower limit on the condenser pressure that can be used. It cannot be lower than the saturation pressure corresponding
to the temperature of the cooling medium. Consider, for example, a condenser that is to be cooled by a nearby river at 15°C. Allowing a temperature difference of 10°C for effective heat transfer, the steam temperature in the condenser must be above 25°C; thus the condenser pressure must be above 3.2 kPa, which is the saturation pressure at 25°C.

**Superheating the Steam to High Temperatures (Increases $T_{\text{high,avg}}$)**

The average temperature at which heat is transferred to steam can be increased without increasing the boiler pressure by superheating the steam to high temperatures. The effect of superheating on the performance of vapor power cycles is illustrated on a $T$-$s$ diagram in Figure (4.7). The colored area on this diagram represents the increase in the net work. The total area under the process curve 3-3' represents the increase in the heat input. Thus both the net work and heat input increase as a result of superheating the steam to a higher temperature. The overall effect is an increase in thermal efficiency, however, since the average temperature at which heat is added increases.

![T-s Diagram](image)

**Figure 4.7** The effect of superheating the steam to higher temperatures on the ideal Rankine cycle.

Superheating the steam to higher temperatures has another very desirable effect: It decreases the moisture content of the steam at the turbine exit, as can be seen from the $T$-$s$ diagram (the quality at state 4' is higher than that at state 4).

The temperature to which steam can be superheated is limited, however, by metallurgical considerations. Presently the highest steam temperature allowed at the turbine inlet is about 620°C.
(1150°F). Any increase in this value depends on improving the present materials or finding new ones that can withstand higher temperatures. Ceramics are very promising in this regard.

**Increasing the Boiler Pressure (Increases $T_{\text{high,avg}}$)**

Another way of increasing the average temperature during the heat-addition process is to increase the operating pressure of the boiler, which automatically raises the temperature at which boiling takes place. This, in turn, raises the average temperature at which heat is transferred to the steam and thus raises the thermal efficiency of the cycle.

The effect of increasing the boiler pressure on the performance of vapor power cycles is illustrated on a $T$-$s$ diagram in Figure (4.8). Notice that for a fixed turbine inlet temperature, the cycle shifts to the left and the moisture content of steam at the turbine exit increases. This undesirable side effect can be corrected, however, by reheating the steam, as discussed in the next section.

![Figure 4.8](image)

*Figure 4.8 The effect of increasing the boiler pressure on the ideal Rankine cycle.*

### 4.5 The Ideal Reheat Rankine Cycle

We noted in the last section that increasing the boiler pressure increases the thermal efficiency of the Rankine cycle, but it also increases the moisture content of the steam to unacceptable levels. There is two possibilities to overcome this problem:

- Superheat the steam to very high temperatures before it enters the turbine. This would be the desirable solution since the average temperature at which heat is added would also
increase, thus increasing the cycle efficiency. This is not a practicable solution, however, since it requires raising the steam temperature to metallurgically unsafe levels.

- Expand the steam in the turbine in two stages, and reheat it in between. In other words, modify the simple ideal Rankine cycle with a reheat process. Reheating is a practical solution to the excessive moisture problem in turbines, and it is commonly used in modern steam power plants.

The $T$-$s$ diagram of the ideal reheat Rankine cycle and the schematic of the power plant operating on this cycle are shown in Figure (4.9).

![Figure 4.9 The ideal reheat Rankine cycle.](image)

The ideal reheat Rankine cycle differs from the simple ideal Rankine cycle in that the expansion process takes place in two stages. In the first stage (the high-pressure turbine), steam is expanded isentropically to an intermediate pressure and sent back to the boiler where it is reheated at constant pressure, usually to the inlet temperature of the first turbine stage. Steam then expands isentropically in the second stage (low-pressure turbine) to the condenser pressure. Thus the total heat input and the total turbine work output for a reheat cycle become

\[
\begin{align*}
q_{in} &= q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4) \quad (4.11) \\
\text{and} \\
\dot{w}_{tur, out} &= \dot{w}_{tur, I} + \dot{w}_{tur, II} = (h_5 - h_4) + (h_5 - h_6) \quad (4.12)
\end{align*}
\]

The incorporation of the single reheat in a modern power plant improves the cycle efficiency by 4 to 5 percent by increasing the average temperature at which heat is transferred to the steam.
The average temperature during the reheat process can be increased by increasing the number of expansion and reheat stages. As the number of stages is increased, the expansion and reheat processes approach an isothermal process at the maximum temperature.

The use of more than two reheat stages, however, is not practical. The theoretical improvement in efficiency from the second reheat is about half of that which results from a single reheat. If the turbine inlet pressure is not high enough, double reheat would result in superheated exhaust. This is undesirable as it would cause the average temperature for heat rejection to increase and thus the cycle efficiency to decrease. Therefore, double reheat is used only on supercritical-pressure ($P > 22.06 \text{ MPa}$) power plants. A third reheat stage would increase the cycle efficiency by about half of the improvement attained by the second reheat. This gain is too small to justify the added cost and complexity.

### 4.6 The Ideal Regenerative Rankine Cycle

Another commonly used method for increasing the thermal efficiency of vapor power plants is **regenerative feedwater heating**, or simply **regeneration**.

A practical regeneration process in steam power plants is accomplished by extracting, or “bleeding,” steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feedwater instead. The device where the feedwater is heated by regeneration is called a **regenerator**, or a **feedwater heater (FWH)**.

**Open Feedwater Heaters**

An open (or direct-contact) **feedwater heater** is basically a **mixing chamber**, where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure. The schematic of a steam power plant with one open feedwater heater (also called **single-stage regenerative cycle**) and the $T$-$s$ diagram of the cycle are shown in Figure 4.10.

In an ideal regenerative Rankine cycle, steam enters the turbine at the boiler pressure (state 5) and expands isentropically to an intermediate pressure (state 6). Some steam is extracted at this state and routed to the feedwater heater, while the remaining steam continues to expand isentropically to the condenser pressure (state 7). This steam leaves the condenser as a saturated liquid at the
condenser pressure (state 1). The condensed water, which is also called the *feedwater*, then enters an isentropic pump, where it is compressed to the feedwater heater pressure (state 2) and is routed to the feedwater heater, where it mixes with the steam extracted from the turbine. The fraction of the steam extracted is such that the mixture leaves the heater as a saturated liquid at the heater pressure (state 3). A second pump raises the pressure of the water to the boiler pressure (state 4). The cycle is completed by heating the water in the boiler to the turbine inlet state (state 5).

![Figure 4.10 The ideal regenerative Rankine cycle with an open feedwater heater.](image)

In the analysis of steam power plants, it is more convenient to work with quantities expressed per unit mass of the steam flowing through the boiler. For each 1 kg of steam leaving the boiler, \( y \) kg expands partially in the turbine and is extracted at state 6. The remaining \((1 - y)\) kg expands completely to the condenser pressure. Therefore, the mass flow rates are different in different components. If the mass flow rate through the boiler is \( \dot{m} \), for example, it is \((1 - y)\dot{m}\) through the condenser. This aspect of the regenerative Rankine cycle should be considered in the analysis of the cycle as well as in the interpretation of the areas on the T-s diagram.

\[
q_{in} = h_5 - h_4 \tag{4.13}
\]
\[
q_{out} = (1 - y)(h_7 - h_1) \tag{4.14}
\]
\[
w_{\text{turb,out}} = (h_5 - h_6) + (1 - y)(h_6 - h_7) \tag{4.15}
\]
\[
w_{\text{pump,in}} = (1 - y)w_{\text{pumpI,in}} + w_{\text{pumpII,in}} \tag{4.16}
\]
Where \[ y = \frac{n_{16}}{m_5}\] (fraction of steam extracted)

\[ w_{\text{pump}_1,\text{in}} = v_1(P_2 - P_1) \] (4.17)

\[ w_{\text{pump}_2,\text{in}} = v_3(P_4 - P_3) \] (4.18)

The thermal efficiency of the Rankine cycle increases as a result of regeneration. This is because regeneration raises the average temperature at which heat is transferred to the steam in the boiler by raising the temperature of the water before it enters the boiler. The cycle efficiency increases further as the number of feedwater heaters is increased. Many large plants in operation today use as many as eight feedwater heaters. The optimum number of feedwater heaters is determined from economical considerations. The use of an additional feedwater heater cannot be justified unless it saves more from the fuel costs than its own cost.

**Closed Feedwater Heaters**

Regenerative feedwater heating also can be accomplished with closed feedwater heaters. Closed heaters are shell-and-tube-type recuperators in which the feedwater temperature increases as the extracted steam condenses on the outside of the tubes carrying the feedwater. Since the two streams do not mix, they can be at different pressures.

**Figure 4.11 Examples of closed feedwater heaters.**

The schematic of a steam power plant with one closed feedwater heater and the T-s diagram of the cycle are shown in Figure (4.12). In an ideal closed feedwater heater, the feedwater is heated to the exit temperature of the extracted steam, which ideally leaves the heater as a saturated liquid at the extraction pressure. In actual power plants, the feedwater leaves the heater below the exit
temperature of the extracted steam because a temperature difference of at least a few degrees is required for any effective heat transfer to take place.

![Figure 4.12 The ideal regenerative Rankine cycle with a closed feedwater heater.](image)

The condensed steam is then either pumped to the feedwater line or routed to another heater or to the condenser through a device called a trap. A trap allows the liquid to be throttled to a lower pressure region but traps the vapor. The enthalpy of steam remains constant during this throttling process.

The open and closed feedwater heaters can be compared as follows. Open feedwater heaters are simple and inexpensive and have good heat transfer characteristics. They also bring the feedwater to the saturation state. For each heater, however, a pump is required to handle the feedwater. The closed feedwater heaters are more complex because of the internal tubing network, and thus they are more expensive. Heat transfer in closed feedwater heaters is also less effective since the two streams are not allowed to be in direct contact. However, closed feedwater heaters do not require a separate pump for each heater since the extracted steam and the feedwater can be at different pressures. Most steam power plants use a combination of open and closed feedwater heaters, as shown in Figure (4.13).
4.7 Cogeneration

Cogeneration is the production of more than one useful form of energy (such as process heat and electric power) from the same energy source.

4.8 Combined Gas–vapor Power Cycles
Refrigeration cycles

6.1 Introduction

We all know from experience that heat flows in the direction of decreasing temperature, that is, from high-temperature regions to low-temperature ones. This heat-transfer process occurs in nature without requiring any devices. The reverse process, however, cannot occur by itself. The transfer of heat from a low-temperature region to a high-temperature one requires special devices called refrigerators. Refrigerators are cyclic devices, and the working fluids used in the refrigeration cycles are called refrigerants.

A refrigerator is shown schematically in Fig. 6.1. Here $Q_L$ is the magnitude of the heat removed from the refrigerated space at temperature $T_L$, $Q_H$ is the magnitude of the heat rejected to the warm space at temperature $T_H$, and $W_{\text{net,in}}$ is the net work input to the refrigerator.

Another device that transfers heat from a low-temperature medium to a high-temperature one is the heat pump. Refrigerators and heat pumps are essentially the same devices; they differ in their objectives only. The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. Discharging this heat to a higher-temperature medium is merely a necessary part of the operation, not the purpose. The objective of a heat pump, however, is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from
a low-temperature source, such as well water or cold outside air in winter, and supplying this heat
to a warmer medium such as a house (Fig. 6.2).

\[ \text{Figure 6.2 Heat pump cycle} \]

The performance of refrigerators and heat pumps is expressed in terms of the \textbf{coefficient of performance} (COP), defined as

\[
\begin{align*}
\text{COP}_{R} &= \frac{\text{Required input}}{\text{Cooling effect}} = \frac{Q_L}{W_{\text{net,in}}} & \quad (6.1) \\
\text{COP}_{HP} &= \frac{\text{Required input}}{\text{Heating effect}} = \frac{Q_H}{W_{\text{net,in}}} & \quad (6.2)
\end{align*}
\]

These relations can also be expressed in the rate form by replacing the quantities \(Q_L, Q_H,\) and \(W_{\text{net,in}}\) by \(\dot{Q}_L, \dot{Q}_H,\) and \(\dot{W}_{\text{net,in}},\) respectively. Notice that both \(\text{COP}_{R}\) and \(\text{COP}_{HP}\) can be greater than 1. A comparison of Eqs. 6.1 and 6.2 reveals that

\[
\text{COP}_{HP} = \text{COP}_{R} + 1 \quad (6.3)
\]

The \textit{cooling capacity} of a refrigeration system, that is, the rate of heat removal from the refrigerated space, is often expressed in terms of \textit{tons of refrigeration}. The capacity of a refrigeration system that can freeze 1 ton of liquid water at 0°C (32°F) into ice at 0°C in 24 h is said to be 1 ton. One ton of refrigeration is equivalent to 211 kJ/min. The cooling load of a typical 200 m² residence is in the 3-ton (10-kW) range.
6.2  The Ideal Vapor-Compression Refrigeration Cycle

The vapor-compression refrigeration cycle is the most widely used cycle for refrigerators, air-conditioning systems, and heat pumps. It consists of four processes:

1-2 Isentropic compression in a compressor
2-3 Constant-pressure heat rejection in a condenser
3-4 Throttling in an expansion device
4-1 Constant-pressure heat absorption in an evaporator

In an ideal vapor-compression refrigeration cycle, the refrigerant enters the compressor at state 1 as saturated vapor and is compressed isentropically to the condenser pressure. The temperature of the refrigerant increases during this isentropic compression process to well above the temperature of the surrounding medium. The refrigerant then enters the condenser as superheated vapor at state 2 and leaves as saturated liquid at state 3 as a result of heat rejection to the surroundings. The temperature of the refrigerant at this state is still above the temperature of the surroundings.

![Schematic and T-s diagram for the ideal vapor-compression refrigeration cycle.](image)

The saturated liquid refrigerant at state 3 is throttled to the evaporator pressure by passing it through an expansion valve or capillary tube. The temperature of the refrigerant drops below the temperature of the refrigerated space during this process. The refrigerant enters the evaporator at state 4 as a low-quality saturated mixture, and it completely evaporates by absorbing heat from the...
refrigerated space. The refrigerant leaves the evaporator as saturated vapor and reenters the compressor, completing the cycle.

In a household refrigerator, the tubes in the freezer compartment where heat is absorbed by the refrigerant serve as the evaporator. The coils behind the refrigerator, where heat is dissipated to the kitchen air, serve as the condenser (Fig. 6.4).

*Figure 6.4* An ordinary household refrigerator.

Remember that the area under the process curve on a $T$-$s$ diagram represents the heat transfer for internally reversible processes. The area under the process curve 4-1 represents the heat absorbed by the refrigerant in the evaporator, and the area under the process curve 2-3 represents the heat rejected in the condenser. A rule of thumb is that the *COP improves by 2 to 4 percent for each °C the evaporating temperature is raised or the condensing temperature is lowered.*

**Compression Process 1-2:**

Isentropic compression process of the saturated water from state point 1 to state point 2 results in rise of pressure from $P_1$ to $P_2$. The work of compression is

$$w_c = h_2 - h_1$$

(6.4)

**Condensation Process 2 - 3:**
Superheated vapor at state 2 is cooled at constant pressure $P_2$ in the condenser until saturated liquid at state 3 is obtained. The heat removed from the refrigerant is:

$$q_{2-3} = h_3 - h_2$$  \hspace{1cm} (6.5)

**Throttling Process 3-4:**
Throttling, which is irreversible process, proceeds along the isenthalpic 3-4. The irreversible constant enthalpy process is represented as a dotted line and for an adiabatic expansion process:

$$h_3 = h_4$$  \hspace{1cm} (6.6)

**Evaporating Process 4 - 1:**
The evaporation process is assumed to be a constant pressure process. To make complete use of the latent heat of the refrigerant, evaporation is continued until the vapor is saturated at state point 1. The refrigeration effect is given by:

$$q_{4-1} = h_1 - h_4$$  \hspace{1cm} (6.7)

Another diagram frequently used in the analysis of vapor-compression refrigeration cycles is the $P-h$ diagram, as shown in Fig. 6.5. On this diagram, three of the four processes appear as straight lines, and the heat transfer in the condenser and the evaporator is proportional to the lengths of the corresponding process curves.

All four components associated with the vapor-compression refrigeration cycle are steady-flow devices, and thus all four processes that make up the cycle can be analyzed as steady-flow processes. The kinetic and potential energy changes of the refrigerant are usually small relative to the work and heat transfer terms, and therefore they can be neglected. Then the steady-flow energy equation on a unit–mass basis reduces to

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = h_e - h_i$$  \hspace{1cm} (6.8)
Figure 6.5 The P-h diagram of an ideal vapor-compression refrigeration cycle.

The condenser and the evaporator do not involve any work, and the compressor can be approximated as adiabatic. Then the COPs of refrigerators and heat pumps operating on the vapor-compression refrigeration cycle can be expressed as

\[
COP_r = \frac{q_L}{W_{net,in}} = \frac{h_1 - h_i}{h_2 - h_i} \quad (6.9)
\]

and

\[
COP_{hp} = \frac{q_H}{W_{net,in}} = \frac{h_2 - h_3}{h_2 - h_i} \quad (6.10)
\]

Where \( h_1 = h_{g@P_1} \) and \( h_3 = h_{f@P_3} \) for the ideal case.

6.3 Actual Vapor-Compression Refrigeration Cycle

An actual vapor-compression refrigeration cycle differs from the ideal one in several ways, owing mostly to the irreversibilities that occur in various components. Two common sources of irreversibilities are fluid friction (causes pressure drops) and heat transfer to or from the surroundings. The T-s diagram of an actual vapor-compression refrigeration cycle is shown in Fig. 6.6.

In the ideal cycle, the refrigerant leaves the evaporator and enters the compressor as saturated vapor. In practice, however, it may not be possible to control the state of the refrigerant so precisely. Instead, it is easier to design the system so that the refrigerant is slightly superheated at the compressor inlet. This slight overdesign ensures that the refrigerant is completely vaporized.
when it enters the compressor. Also, the line connecting the evaporator to the compressor is usually very long; thus the pressure drop caused by fluid friction and heat transfer from the surroundings to the refrigerant can be very significant. The result of superheating, heat gain in the connecting line, and pressure drops in the evaporator and the connecting line is an increase in the specific volume, thus an increase in the power input requirements to the compressor since steady-flow work is proportional to the specific volume.

![Schematic and T-s diagram for the actual vapor-compression refrigeration cycle.](image)

**Figure 6.6** Schematic and T-s diagram for the actual vapor-compression refrigeration cycle.

The compression process in the ideal cycle is internally reversible and adiabatic, and thus isentropic. The actual compression process, however, involves frictional effects, which increase the entropy, and heat transfer, which may increase or decrease the entropy, depending on the direction. Therefore, the entropy of the refrigerant may increase (process 1-2) or decrease (process 1-2) during an actual compression process, depending on which effects dominate. The compression process 1-2 may be even more desirable than the isentropic compression process since the specific volume of the refrigerant and thus the work input requirement are smaller in this case. Therefore, the refrigerant should be cooled during the compression process whenever it is practical and economical to do so.

In the ideal case, the refrigerant is assumed to leave the condenser as saturated liquid at the compressor exit pressure. In reality, however, it is unavoidable to have some pressure drop in the condenser as well as in the lines connecting the condenser to the compressor and to the throttling valve. Also, it is not easy to execute the condensation process with such precision that the refrigerant is a saturated liquid at the end, and it is undesirable to route the refrigerant to the
throttling valve before the refrigerant is completely condensed. Therefore, the refrigerant is subcooled somewhat before it enters the throttling valve. We do not mind this at all, however, since the refrigerant in this case enters the evaporator with a lower enthalpy and thus can absorb more heat from the refrigerated space. The throttling valve and the evaporator are usually located very close to each other, so the pressure drop in the connecting line is small.

### 6.4 Innovative Vapor-Compression Refrigeration Systems

The simple vapor-compression refrigeration cycle discussed above is the most widely used refrigeration cycle, and it is adequate for most refrigeration applications. The ordinary vapor-compression refrigeration systems are simple, inexpensive, reliable, and practically maintenance-free (when was the last time you serviced your household refrigerator?). However, for large industrial applications *efficiency*, not simplicity, is the major concern. Also, for some applications the simple vapor-compression refrigeration cycle is inadequate and needs to be modified. We now discuss a few such modifications and refinements.

#### 6.4.1 Effect of Sub-cooling the Refrigerant

A simple vapor-compression refrigeration cycle with sub-cooling shown in Fig. 6.4 When the liquid refrigerant is sub-cooled from 3' - 3, the expansion process takes place along 3 - 4 instead of 3' - 4'. The refrigeration effect increases from \((h_1 - h_4')\), to \((h_1 - h_4)\) by sub-cooling, the extra refrigeration effect being \((h_4' - h_4)\). The work of compression is considered the same for both the simple saturated cycle as well as the sub-cooled cycle. As no extra work is required by the compressor, the net effect of sub-cooling the refrigerant is to increase the COP of the refrigerator.
Sub-cooling, that is increasing the refrigeration effect of the system, reduces the mass flow rate of the refrigerant for a system having the same capacity as the simple vapor-compression refrigeration system.

### 6.4.2 Effect of Superheating the Refrigerant

In the ideal simple saturated vapor-compression refrigeration cycle, the refrigerant leaves the evaporator is assumed to be saturated. In practice, however, the evaporation process is continued until the refrigerant is superheated by about 5°C to 20°C. The superheat may be created by any of the following processes.

- If the evaporator load is large enough, the vapor may be superheated.
- If heat exchange occurs between liquid from the condenser and vapor leaving the evaporator, then the vapor may be superheated.
- Heat transfer from the surroundings may superheat vapor leaving the evaporator.

Superheating the refrigerant increases the net mechanical work input to achieve the same compression ratio and reduces the COP of the refrigerating plant for the same capacity of the system.

![Figure 6.8 Superheating the Refrigerant](image)

### 6.4.3 Cascade Refrigeration Systems

Some industrial applications require moderately low temperatures, and the temperature range they involve may be too large for a single vapor-compression refrigeration cycle to be practical. A large temperature range also means a large pressure range in the cycle and a poor performance for a
reciprocating compressor. One way of dealing with such situations is to perform the refrigeration process in stages, that is, to have two or more refrigeration cycles that operate in series. Such refrigeration cycles are called **cascade refrigeration cycles**.

A two-stage cascade refrigeration cycle is shown in Fig. 6.7. The two cycles are connected through the heat exchanger in the middle, which serves as the evaporator for the topping cycle (cycle $A$) and the condenser for the bottoming cycle (cycle $B$). Assuming the heat exchanger is well insulated and the kinetic and potential energies are negligible, the heat transfer from the fluid in the bottoming cycle should be equal to the heat transfer to the fluid in the topping cycle. Thus, the ratio of mass flow rates through each cycle should be

$$\frac{m_A}{m_B} = \frac{h_2 - h_3}{h_z - h_b}$$

(6.12)

Also

$$\text{COP}_{R,\text{cascade}} = \frac{Q_L}{W_{\text{net,in}}} = \frac{m_B(h_1 - h_4)}{m_A(h_b - h_z) + m_B(h_z - h_b)}$$

(6.13)

---

**Figure 6.9** A two-stage cascade refrigeration system with the same refrigerant in both stages.
In the cascade system shown in the figure, the refrigerants in both cycles are assumed to be the same. This is not necessary, however, since there is no mixing taking place in the heat exchanger. Therefore, refrigerants with more desirable characteristics can be used in each cycle. In this case, there would be a separate saturation dome for each fluid, and the $T$-$s$ diagram for one of the cycles would be different. Also, in actual cascade refrigeration systems, the two cycles would overlap somewhat since a temperature difference between the two fluids is needed for any heat transfer to take place.

It is evident from the $T$-$s$ diagram in Fig. 6.7 that the compressor work decreases and the amount of heat absorbed from the refrigerated space increases as a result of cascading. Therefore, cascading improves the COP of a refrigeration system. Some refrigeration systems use three or four stages of cascading.

### 6.4.4 Multistage Compression Refrigeration Systems

When the fluid used throughout the cascade refrigeration system is the same, the heat exchanger between the stages can be replaced by a mixing chamber (called a *flash chamber*) since it has better heat transfer characteristics. Such systems are called *multistage compression refrigeration systems*. A two stage compression refrigeration system is shown in Fig. 6.8.

![Figure 6.10 A two-stage compression refrigeration system with a flash chamber.](image)

In this system, the liquid refrigerant expands in the first expansion valve to the flash chamber pressure, which is the same as the compressor interstage pressure. Part of the liquid vaporizes...
during this process. This saturated vapor (state 3) is mixed with the superheated vapor from the
downstream compressor (state 2), and the mixture enters the high-pressure compressor at state 9.
This is, in essence, a regeneration process. The saturated liquid (state 7) expands through the
second expansion valve into the evaporator, where it picks up heat from the refrigerated space.
The compression process in this system resembles a two-stage compression with intercooling, and
the compressor work decreases. Care should be exercised in the interpretations of the areas on the
$T-s$ diagram in this case since the mass flow rates are different in different parts of the cycle.

### 6.4.5 Air Refrigeration cycle

Air refrigeration cycle is basically similar to the vapor-compression cycle, the exception being in the
expansion process. For air refrigeration cycles, the expansion process is provided by means of a
turbine which replaces the expansion valve of the - compression refrigeration cycles. When air
which is treated as an ideal gas expands through the turbine, its temperature is reduced.
In practice the air refrigeration cycle is commonly used for air conditioning aircraft in an open
cycle. The use of air refrigeration system has the following advantages.

- Air is easily available and is cheap compared with other refrigerants
- Air is non-flammable and no danger of fire
- The mass of air refrigeration system per ton of refrigeration is low compared with the other
  refrigeration system.

A simple air refrigeration plant arrangement is represented as schematic diagram and on a T-s
diagram in Fig. 6.11.

The COP of the air refrigerant cycle,

\[
COP_R = \frac{q_L}{w_{net,in}} = \frac{q_L}{w_{comp,in} - w_{turb,out}}
\]  

(6.14)

Where:-

\[
q_L = h_i - h_4 \quad \text{(6.15)}
\]

\[
w_{turb,out} = h_3 - h_4 \quad \text{(6.16)}
\]

\[
w_{comp,in} = h_2 - h_i \quad \text{(6.17)}
\]
Figure 6.11 Air Refrigeration cycle
Chapter 5
Air-Standard Power Cycle

6.5 Introduction

In this chapter engines that utilized gas as working fluid are introduced. In gas power cycles, the working fluid remains a gas throughout the entire cycle. Spark-ignition engines, diesel engines, and conventional gas turbines are familiar examples of devices that operate on gas cycles. In all these engines, energy is provided by burning a fuel within the system boundaries. That is, they are internal combustion engines. Because of this combustion process, the composition of the working fluid changes from air and fuel to combustion products during the course of the cycle.

Even though internal combustion engines operate on a mechanical cycle (the piston returns to its starting position at the end of each revolution), the working fluid does not undergo a complete thermodynamic cycle. It is thrown out of the engine at some point in the cycle (as exhaust gases) instead of being returned to the initial state. Working on an open cycle is the characteristic of all internal combustion engines.

The actual gas power cycles are rather complex. To reduce the analysis to a manageable level, we utilize the following approximations, commonly known as the air-standard assumptions:

- The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
- All the processes that make up the cycle are internally reversible.
- The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.
- The combustion process is replaced by a heat-addition process from an external source.

![Figure 5.1 The combustion process is replaced by a heat-addition process in ideal cycles.](image)
Another assumption that is often utilized to simplify the analysis even more is that air has constant specific heats whose values are determined at *room temperature* (25°C, or 77°F). When this assumption is utilized, the air-standard assumptions are called the **cold-air-standard assumptions**.

A cycle for which the air-standard assumptions are applicable is frequently referred to as an **air-standard cycle**. The air-standard assumptions previously stated provide considerable simplification in the analysis without significantly deviating from the actual cycles. This simplified model enables us to study qualitatively the influence of major parameters on the performance of the actual engines.

### 6.6 An Overview of Reciprocating Engines

Despite its simplicity, the reciprocating engine (basically a piston–cylinder device) is one of the rare inventions that has proved to be very versatile and to have a wide range of applications. It is the powerhouse of the vast majority of automobiles, trucks, light aircraft, ships, and electric power generators, as well as many other devices.

The basic components of a reciprocating engine are shown in Figure (5.2). The piston reciprocates in the cylinder between two fixed positions called the **top dead center** (TDC), the position of the piston when it forms the smallest volume in the cylinder and the **bottom dead center** (BDC), the position of the piston when it forms the largest volume in the cylinder. The distance between the TDC and the BDC is the largest distance that the piston can travel in one direction, and it is called the **stroke** of the engine. The diameter of the piston is called the **bore**. The air or air–fuel mixture is drawn into the cylinder through the **intake valve**, and the combustion products are expelled from the cylinder through the **exhaust valve**.
**Figure 5.2** Nomenclature for reciprocating engines.
The minimum volume formed in the cylinder when the piston is at TDC is called the *clearance volume* (Figure 5.3).

![Diagram](image1)

**Figure 5.3** Displacement and clearance volumes of a reciprocating engine.
The volume displaced by the piston as it moves between TDC and BDC is called the *displacement volume*. The ratio of the maximum volume formed in the cylinder to the minimum (clearance) volume is called the *compression ratio* $r$ of the engine.

$$r = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}}$$  \hspace{1cm} (5.1)

Another term frequently used in conjunction with reciprocating engines is the *mean effective pressure* (MEP). It is a fictitious pressure that, if it acted on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle (Figure 5.4). That is,
**Figure 5.4** The net work output of a cycle is equivalent to the product of the mean effective pressure and the displacement volume.

\[ W_{\text{net}} = \text{MEP} \times \text{Piston area} \times \text{stroke} = \text{MEP} \times \text{Displacement volume} \]

Or

\[ \text{MEP} = \frac{W_{\text{net}}}{V_{\max} - V_{\min}} = \frac{W_{\text{ney}}}{V_{\max} - V_{\min}} \quad \text{(kPa)} \quad (5.2) \]

The mean effective pressure can be used as a parameter to compare the performances of reciprocating engines of equal size. The engine with a larger value of MEP delivers more net work per cycle and thus performs better.

Reciprocating engines are classified as **spark-ignition (SI) engines** or **compression-ignition (CI) engines**, depending on how the combustion process in the cylinder is initiated. In SI engines, the combustion of the air-fuel mixture is initiated by a spark plug. In CI engines, the air-fuel mixture is self-ignited as a result of compressing the mixture above its self-ignition temperature. In the next two sections, we discuss the **Otto and Diesel cycles**, which are the ideal cycles for the SI and CI reciprocating engines, respectively.

### 6.7 Air-Standard Otto Cycle

The Otto cycle is the ideal cycle for spark-ignition reciprocating engines. It is named after Nikolaus A. Otto, who built a successful four-stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862.

In most spark-ignition engines, the piston executes four complete strokes (two mechanical cycles) within the cylinder, and the crankshaft completes two revolutions for each thermodynamic cycle. These engines are called **four-stroke** internal combustion engines. A schematic of each stroke as well as a P-v diagram for an actual four-stroke spark-ignition engine is given in Figure (5.5).
Figure 5.5 Actual and ideal cycles in spark-ignition engines and their P-v diagrams.

Initially, both the intake and the exhaust valves are closed, and the piston is at its lowest position (BDC). During the compression stroke, the piston moves upward, compressing the air-fuel mixture. Shortly before the piston reaches its highest position (TDC), the spark plug fires and the mixture ignite, increasing the pressure and temperature of the system. The high-pressure gases force the piston down, which in turn forces the crankshaft to rotate, producing a useful work output during the expansion or power stroke. At the end of this stroke, the piston is at its lowest position (the completion of the first mechanical cycle), and the cylinder is filled with combustion products. Now the piston moves upward one more time, purging the exhaust gases through the exhaust valve (the exhaust stroke), and down a second time, drawing in fresh air-fuel mixture through the intake valve (the intake stroke). Notice that the pressure in the cylinder is slightly above the atmospheric value during the exhaust stroke and slightly below during the intake stroke.

In two-stroke engines, all four functions described above are executed in just two strokes: the power stroke and the compression stroke. In these engines, the crankcase is sealed, and the outward
motion of the piston is used to slightly pressurize the air–fuel mixture in the crankcase, as shown in Figure (5.6). Also, the intake and exhaust valves are replaced by openings in the lower portion of the cylinder wall. During the latter part of the power stroke, the piston uncovers first the exhaust port, allowing the exhaust gases to be partially expelled, and then the intake port, allowing the fresh air–fuel mixture to rush in and drive most of the remaining exhaust gases out of the cylinder. This mixture is then compressed as the piston moves upward during the compression stroke and is subsequently ignited by a spark plug.

![Figure 5.6 Schematic of a two-stroke reciprocating engine.](image)

The two-stroke engines are generally less efficient than their four-stroke counterparts because of the incomplete expulsion of the exhaust gases and the partial expulsion of the fresh air–fuel mixture with the exhaust gases. However, they are relatively simple and inexpensive, and they have high power-to-weight and power-to-volume ratios, which make them suitable for applications requiring small size and weight such as for motorcycles, chain saws, and lawn mowers.

The thermodynamic analysis of the actual four-stroke or two-stroke cycles described is not a simple task. However, the analysis can be simplified significantly if the air-standard assumptions are utilized. The resulting cycle, which closely resembles the actual operating conditions, is the ideal **Otto cycle**.

The air-standard Otto cycle is an ideal cycle that assumes the heat addition occurs instantaneously while the piston is at top dead center. The **Otto cycle** is shown on the $p$-$v$ and $T$-$s$
Figure 5.7 p–v and T–s diagrams of the air-standard Otto cycle.

It consists of four internally reversible processes:

- Process 1–2 is an isentropic compression of the air as the piston moves from bottom dead center to top dead center.
- Process 2–3 is a constant-volume heat transfer to the air from an external source while the piston is at top dead center. This process is intended to represent the ignition of the fuel–air mixture and the subsequent rapid burning.
- Process 3–4 is an isentropic expansion (power stroke).
- Process 4–1 completes the cycle by a constant-volume process in which heat is rejected from the air while the piston is at bottom dead center.

The Otto cycle is executed in a closed system, and disregarding the changes in kinetic and potential energies, the energy balance for any of the processes is expressed, on a unit-mass basis, as

$$\Delta u = (q_{in} - q_{out}) + (w_{in} - w_{out})$$ (5.3)

No work is involved during the two heat transfer processes since both take places at constant volume. Therefore, heat transfer to and from the working fluid can be expressed as

$$q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$ (5.4)

and

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$ (5.5)

Then the thermal efficiency of the ideal Otto cycle under the cold air standard assumptions becomes

$$\eta_{th, Otto} = \frac{w_{net}}{q_{in}}$$ (5.6)

but

$$w_{net} = q_{in} - q_{out}$$ (5.7)
\[
\eta_{th,\text{Otto}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_3 - 1)}{T_2(T_4/T_3 - 1)} \tag{5.8}
\]

Processes 1-2 and 3-4 are isentropic, and \(v_2 = v_3\) and \(v_4 = v_1\). Thus,

\[
\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3} \quad \text{thus} \quad \frac{T_2}{T_2} = \frac{T_4}{T_1} \tag{5.9}
\]

Substituting these equations into the thermal efficiency relation and simplifying give

\[
\eta_{th,\text{Otto}} = 1 - \frac{1}{r^{k-1}} \tag{5.10}
\]

Where:

\[
r = \frac{V_{max}}{V_{min}} = \frac{V_4}{V_2} = \frac{v_1}{v_2} \tag{5.11}
\]

is the compression ratio and \(k\) is the specific heat ratio \(c_p/c_v\).

Equation 5.10 shows that under the cold-air-standard assumptions, the thermal efficiency of an ideal Otto cycle depends on the compression ratio of the engine and the specific heat ratio of the working fluid. The thermal efficiency of the ideal Otto cycle increases with both the compression ratio and the specific heat ratio.

A plot of thermal efficiency versus the compression ratio is given in Figure (5.8) for \(k = 1.4\), which is the specific heat ratio value of air at room temperature. For a given compression ratio, the thermal efficiency of an actual spark-ignition engine is less than that of an ideal Otto cycle because of the irreversibilities, such as friction, and other factors such as incomplete combustion.

**Figure 5.8** Thermal efficiency of the ideal Otto cycle as a function of compression ratio\((k = 1.4)\). We can observe from Figure 5.8 that the thermal efficiency curve is rather steep at low compression ratios but flattens out starting with a compression ratio value of about 8. Therefore,
the increase in thermal efficiency with the compression ratio is not as pronounced at high compression ratios. Also, when high compression ratios are used, the temperature of the air–fuel mixture rises above the autoignition temperature of the fuel (the temperature at which the fuel ignites without the help of a spark) during the combustion process, causing an early and rapid burn of the fuel at some point or points ahead of the flame front, followed by almost instantaneous inflammation of the end gas. This premature ignition of the fuel, called autoignition, produces an audible noise, which is called engine knock. Autoignition in spark-ignition engines cannot be tolerated because it hurts performance and can cause engine damage. The requirement that autoignition not be allowed places an upper limit on the compression ratios that can be used in spark-ignition internal combustion engines.

The second parameter affecting the thermal efficiency of an ideal Otto cycle is the specific heat ratio $k$. For a given compression ratio, an ideal Otto cycle using a monatomic gas (such as argon or helium, $k = 1.667$) as the working fluid will have the highest thermal efficiency. The specific heat ratio $k$, and thus the thermal efficiency of the ideal Otto cycle, decreases as the molecules of the working fluid get larger (Figure 5.9). At room temperature it is 1.4 for air, 1.3 for carbon dioxide, and 1.2 for ethane. The working fluid in actual engines contains larger molecules such as carbon dioxide, and the specific heat ratio decreases with temperature, which is one of the reasons that the actual cycles have lower thermal efficiencies than the ideal Otto cycle. The thermal efficiencies of actual spark-ignition engines range from about 25 to 30 percent.

![Figure 5.9](image)

**Figure 5.9** The thermal efficiency of the Otto cycle increases with the specific heat ratio $k$ of the working fluid.

### 6.8 Air-Standard Diesel Cycle
The Diesel cycle is the ideal cycle for CI reciprocating engines. The CI engine, first proposed by Rudolph Diesel in the 1890s, is very similar to the SI engine, differing mainly in the method of initiating combustion. In spark-ignition engines (also known as gasoline engines), the air–fuel mixture is compressed to a temperature that is below the autoignition temperature of the fuel, and the combustion process is initiated by firing a spark plug. In CI engines (also known as diesel engines), the air is compressed to a temperature that is above the autoignition temperature of the fuel, and combustion starts on contact as the fuel is injected into this hot air. Therefore, the spark plug and carburetor are replaced by a fuel injector in diesel engines (Figure 5.10).

![Figure 5.10 Gasoline and Diesel engine ignition system](image)

In gasoline engines, a mixture of air and fuel is compressed during the compression stroke, and the compression ratios are limited by the onset of autoignition or engine knock. In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of autoignition. Therefore, diesel engines can be designed to operate at much higher compression ratios, typically between 12 and 24. Not having to deal with the problem of autoignition has another benefit: many of the stringent requirements placed on the gasoline can now be removed, and fuels that are less refined (thus less expensive) can be used in diesel engines.

The fuel injection process in diesel engines starts when the piston approaches TDC and continues during the first part of the power stroke. Therefore, the combustion process in these engines takes place over a longer interval. Because of this longer duration, the combustion process in the ideal Diesel cycle is approximated as a constant-pressure heat-addition process. In fact, this is the only process where the Otto and the Diesel cycles differ. The remaining three processes are the same.
for both ideal cycles. That is, process 1-2 is isentropic compression, 3-4 is isentropic expansion, and 4-1 is constant-volume heat rejection. The similarity between the two cycles is also apparent from the $P$-$v$ and $T$-$s$ diagrams of the Diesel cycle, shown in Figure (5.11).

![Figure 5.11 T-s and P-v diagrams for the ideal Diesel cycle.](image)

Noting that the Diesel cycle is executed in a piston-cylinder device, which forms a closed system, the amount of heat transferred to the working fluid at constant pressure and rejected from it at constant volume can be expressed as

$$q_{in} - w_{h, out} = u_3 - u_2$$  \hspace{1cm} (5.12)

$$q_{in} = P_2 (v_3 - v_2) + (u_3 - u_2)$$  \hspace{1cm} (5.13)

$$q_{in} = h_3 - h_2 = c_p (T_3 - T_2)$$  \hspace{1cm} (5.14)

and

$$q_{out} = u_4 - u_1 = c_v (T_4 - T_1)$$  \hspace{1cm} (5.15)

Then the thermal efficiency of the ideal Diesel cycle under the cold-air-standard assumptions becomes

$$\eta_{th, Diesel} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{\frac{T_4}{T_2} / \frac{T_3}{T_2} - 1}{k(T_3 / T_2 - 1)}$$  \hspace{1cm} (5.16)

We now define a new quantity, the cutoff ratio $r_c$, as the ratio of the cylinder volumes after and before the combustion process:

$$r_c = \frac{v_3}{v_2} = \frac{V_3 / V_2}{v_3 / v_2}$$  \hspace{1cm} (5.17)

Utilizing this definition and the isentropic ideal-gas relations for processes 1-2 and 3-4, we see that the thermal efficiency relation reduces to
where $r$ is the compression ratio defined by Equation (5.11). Looking at Equation (5.18) carefully, one would notice that under the cold-air-standard assumptions, the efficiency of a Diesel cycle differs from the efficiency of an Otto cycle by the quantity in the brackets. This quantity is always greater than 1. Therefore,

$$\eta_{th, Otto} > \eta_{th, Diesel}$$

### 6.9 Air-Standard Dual Cycle

Approximating the combustion process in internal combustion engines as a constant-volume or a constant-pressure heat-addition process is overly simplistic and not quite realistic. Probably a better (but slightly more complex) approach would be to model the combustion process in both gasoline and diesel engines as a combination of two heat-transfer processes, one at constant volume and the other at constant pressure. The ideal cycle based on this concept is called the dual cycle, and a $P$-$V$ diagram for it is given in Figure (5.12).

![Figure 5.12 p–v and T–s diagrams of the air-standard dual cycle.](image)

As in the Otto and Diesel cycles, Process 1-2 is an isentropic compression. The heat addition occurs in two steps, however: Process 2-3 is a constant-volume heat addition; Process 3-4 is a constant pressure heat addition. Process 3-4 also makes up the first part of the power stroke. The isentropic expansion from state 4 to state 5 is the remainder of the power stroke. As in the Otto and Diesel
cycles, the cycle is completed by a constant volume heat rejection process, Process 5-1. Areas on the $T-s$ and $p-v$ diagrams can be interpreted as heat and work, respectively, as in the cases of the Otto and Diesel cycles.

### 6.10 Stirling and Ericsson Cycles

Cycle which have efficiencies equal to the Carnot cycle and called Stirling and Ericsson cycles have been defined. Although these cycles are not extensively used to model actual engines, they are presented here to illustrate the effective use of a regenerator—a heat exchanger which utilizes waste heat. In both the Stirling cycles as well as the Ericsson cycles, compression and expansion processes are isothermal. The Stirling and Ericsson cycles are represented schematically figure below.

![Figure 5.13 Stirling and Ericsson cycle](image)

For an ideal regenerator, the heat supplied to the working fluid, process 2-3, is equal to the heat rejection from the working fluid, process 4-1. The isothermal process 1-2 and 3-4 are connected by constant volume and constant pressure process for the Stirling and Ericsson cycle, respectively.

1-2 $T = \text{constant}$ expansion (heat addition from the external source)

2-3 $v = \text{constant}$ regeneration (internal heat transfer from the working fluid to the regenerator)
3-4 \( T = \text{constant} \) compression (heat rejection to the external sink)
4-1 \( v = \text{constant} \) regeneration (internal heat transfer from the regenerator back to the working fluid)

![T-s and P-v diagrams of Carnot, Stirling, and Ericsson cycles.](image)

**Figure 5.14** T-s and P-v diagrams of Carnot, Stirling, and Ericsson cycles.

Both the Stirling and Ericsson cycles are totally reversible, as is the Carnot cycle, and thus according to the Carnot principle, all three cycles must have the same thermal efficiency when operating between the same temperature limits:

\[
\eta_{\text{th,Stirling}} = \eta_{\text{th,Ericsson}} = \eta_{\text{th,Carnot}} = 1 - \frac{T_L}{T_H}
\]

### 6.11 Modeling Gas Turbine Power Plants

Gas turbine power plants may operate on either an open or closed basis. The open mode pictured in Fig. 5.13a is more common. This is an engine in which atmospheric air is continuously drawn into the compressor, where it is compressed to a high pressure. The air then enters a combustion chamber, or combustor, where it is mixed with fuel and combustion occurs, resulting in combustion products at an elevated temperature. The combustion products expand through the turbine and are subsequently discharged to the surroundings. Part of the turbine work developed
is used to drive the compressor; the remainder is available to generate electricity, to propel a vehicle, or for other purposes. In the system pictured in Fig. 5.13b, the working fluid receives an energy input by heat transfer from an external source, for example a gas-cooled nuclear reactor. The gas exiting the turbine is passed through a heat exchanger, where it is cooled prior to reentering the compressor.

Figure 5.15 Simple gas turbine. (a) Open to the atmosphere. (b) Closed.

An idealization often used in the study of open gas turbine power plants is that of an air-standard analysis. In an air-standard analysis two assumptions are always made:

- The working fluid is air, which behaves as an ideal gas.
- The temperature rise that would be brought about by combustion is accomplished by a heat transfer from an external source.

### 6.12 Brayton Cycle: The Ideal Cycle For Gas-Turbine Engines

The Brayton cycle was first proposed by George Brayton for use in the reciprocating oil-burning engine that he developed around 1870. Today, it is used for gas turbines only where both the compression and expansion processes take place in rotating machinery. Gas turbines usually operate on an open cycle, as shown in Fig. 5.13a. Fresh air at ambient conditions is drawn into the compressor, where its temperature and pressure are raised. The high-pressure air proceeds into the combustion chamber, where the fuel is burned at constant pressure. The resulting high-temperature
gases then enter the turbine, where they expand to the atmospheric pressure while producing power. The exhaust gases leaving the turbine are thrown out (not recirculated), causing the cycle to be classified as an open cycle.

The open gas-turbine cycle described above can be modeled as a **closed cycle**, as shown in Fig. 5.13b, by utilizing the air-standard assumptions. Here the compression and expansion processes remain the same, but the combustion process is replaced by a constant-pressure heat-addition process from an external source, and the exhaust process is replaced by a constant-pressure heat-rejection process to the ambient air. The ideal cycle that the working fluid undergoes in this closed loop is the **Brayton Cycle**, which is made up of four internally reversible processes:

1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection

The $T$-$s$ and $P$-$v$ diagrams of an ideal Brayton cycle are shown in Fig. 5–14. Notice that all four processes of the Brayton cycle are executed in steadyflow devices; thus, they should be analyzed as steady-flow processes.

![Figure 5.16 T-s and P-v diagrams for the ideal Brayton cycle.](image)

When the changes in kinetic and potential energies are neglected, the energy balance for a steady-flow process can be expressed, on a unit–mass basis, as

\[
(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}
\]
Therefore, heat transfers to and from the working fluid are
\[ q_{in} = h_3 - h_2 = c_p(T_3 - T_2) \]
and
\[ q_{out} = h_4 - h_1 = c_p(T_4 - T_1) \]
Then the thermal efficiency of the ideal Brayton cycle under the cold-airstandard assumptions becomes
\[ \eta_{th,brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_5 / T_2 - 1)} \]
Processes 1-2 and 3-4 are isentropic, and \( P_2 = P_3 \) and \( P_4 = P_1 \). Thus,
\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = \left( \frac{P_3}{P_1} \right)^{(k-1)/k} = \frac{T_3}{T_4} \]
Substituting these equations into the thermal efficiency relation and simplifying give
\[ \eta_{th,brayton} = 1 - \frac{1}{r_p^{(k-1)/k}} \]
Were
\[ r_p = \frac{P_2}{P_1} \]
is the pressure ratio and \( k \) is the specific heat ratio.

**Deviation of Actual Gas-Turbine Cycles from Idealized Ones**

The actual gas-turbine cycle differs from the ideal Brayton cycle on several accounts. For one thing, some pressure drop during the heat-addition and heat-rejection processes is inevitable. More importantly, the actual work input to the compressor is more, and the actual work output from the turbine is less because of irreversibilities. The deviation of actual compressor and turbine behavior from the idealized isentropic behavior can be accurately accounted for by utilizing the isentropic efficiencies of the turbine and compressor as
\[ \eta_C = \frac{w_s}{w_a} \approx \frac{h_{2a} - h_1}{h_{2a} - h_i} \]
and
Figure 5.17 The deviation of an actual gas-turbine cycle from the ideal Brayton cycle as a result of irreversibilities.

where states 2a and 4a are the actual exit states of the compressor and the turbine, respectively, and 2s and 4s are the corresponding states for the isentropic case, as illustrated in Fig. 5.17. The effect of the turbine and compressor efficiencies on the thermal efficiency of the gas-turbine engines is illustrated below with an example.

6.13 The Brayton Cycle With Regeneration

In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor. Therefore, the high-pressure air leaving the compressor can be heated by transferring heat to it from the hot exhaust gases in a counter-flow heat exchanger, which is also known as a regenerator or a recuperator. A sketch of the gas-turbine engine utilizing a regenerator and the T-s diagram of the new cycle are shown in Figs. 9–38 and 9–39, respectively.

The thermal efficiency of the Brayton cycle increases as a result of regeneration since the portion of energy of the exhaust gases that is normally rejected to the surroundings is now used to preheat the air entering the combustion chamber. This, in turn, decreases the heat input (thus fuel) requirements for the same net work output. Note, however, that the use of a regenerator is recommended only when the turbine exhaust temperature is higher than the compressor exit
temperature. Otherwise, heat will flow in the reverse direction (to the exhaust gases), decreasing the efficiency. This situation is encountered in gas-turbine engines operating at very high pressure ratios.

**Figure 5.18** A gas-turbine engine with regenerator.

The highest temperature occurring within the regenerator is $T_4$, the temperature of the exhaust gases leaving the turbine and entering the regenerator. Under no conditions can the air be preheated in the regenerator to a temperature above this value. Air normally leaves the regenerator at a lower temperature, $T_5$. In the limiting (ideal) case, the air exits the regenerator at the inlet temperature of the exhaust gases $T_4$. Assuming the regenerator to be well insulated and any changes in kinetic and potential energies to be negligible, the actual and maximum heat transfers from the exhaust gases to the air can be expressed as

$$q_{\text{regen,act}} = h_5 - h_2$$
and

\[ q_{\text{regen, max}} = h_5 - h_2 = h_4 - h_2 \]

The extent to which a regenerator approaches an ideal regenerator is called the \textit{effectiveness} and is defined as

\[ \varepsilon = \frac{q_{\text{regen, act}}}{q_{\text{regen, max}}} = \frac{h_5 - h_2}{h_4 - h_2} \]

When the cold-air-standard assumptions are utilized, it reduces to

\[ \varepsilon \approx \frac{T_1 - T_2}{T_4 - T_2} \]

Under the cold-air-standard assumptions, the thermal efficiency of an ideal Brayton cycle with regeneration is

\[ \eta_{\text{th, regen}} = 1 - \left( \frac{T_2}{T_3} \right) \left( r_p \right)^{(k-1)/k} \]

\section*{6.14 The Brayton Cycle with Inter-cooling, Reheating, and Regeneration}

The net work of a gas-turbine cycle is the difference between the turbine work output and the compressor work input, and it can be increased by either decreasing the compressor work or increasing the turbine work, or both. The work required to compress a gas between two specified pressures can be decreased by carrying out the compression process in stages and cooling the gas in between that is, using \textit{multistage compression with inter-cooling}. 
Figure 5.20 Comparison of work inputs to a single-stage compressor (1AC) and a two-stage compressor with inter-cooling (1ABD).

As the number of stages is increased, the compression process becomes nearly isothermal at the compressor inlet temperature, and the compression work decreases.

Likewise, the work output of a turbine operating between two pressure levels can be increased by expanding the gas in stages and reheating it in between—that is, utilizing multistage expansion with reheating. This is accomplished without raising the maximum temperature in the cycle. As the number of stages is increased, the expansion process becomes nearly isothermal. The foregoing argument is based on a simple principle: The steady-flow compression or expansion work is proportional to the specific volume of the fluid. Therefore, the specific volume of the working fluid should be as low as possible during a compression process and as high as possible during an expansion process. This is precisely what inter-cooling and reheating accomplish.

The working fluid leaves the compressor at a lower temperature, and the turbine at a higher temperature, when intercooling and reheating are utilized. This makes regeneration more attractive since a greater potential for regeneration exists. Also, the gases leaving the compressor can be heated to a higher temperature before they enter the combustion chamber because of the higher temperature of the turbine exhaust.
Figure 5.21 A gas-turbine engine with two-stage compression with intercooling, two-stage expansion with reheating, and regeneration.

The gas enters the first stage of the compressor at state 1, is compressed isentropically to an intermediate pressure $P_2$, is cooled at constant pressure to state 3 ($T_3 = T_1$), and is compressed in the second stage isentropically to the final pressure $P_4$. At state 4 the gas enters the regenerator, where it is heated to $T_5$ at constant pressure. In an ideal regenerator, the gas leaves the regenerator at the temperature of the turbine exhaust, that is, $T_5 - T_9$. The primary heat addition (or combustion) process takes place between states 5 and 6. The gas enters the first stage of the turbine at state 6 and expands isentropically to state 7, where it enters the reheater. It is reheated at constant pressure to state 8 ($T_8 = T_6$), where it enters the second stage of the turbine. The gas exits the turbine at state...
9 and enters the regenerator, where it is cooled to state 10 at constant pressure. The cycle is completed by cooling the gas to the initial state (or purging the exhaust gases).

### 6.15 Ideal Jet-Propulsion Cycles

Gas-turbine engines are widely used to power aircraft because they are light and compact and have a high power-to-weight ratio. Aircraft gas turbines operate on an open cycle called a jet-propulsion cycle. The ideal jet-propulsion cycle differs from the simple ideal Brayton cycle in that the gases are not expanded to the ambient pressure in the turbine. Instead, they are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor and the auxiliary equipment, such as a small generator and hydraulic pumps. That is, the net work output of a jet-propulsion cycle is zero. The gases that exit the turbine at a relatively high pressure are subsequently accelerated in a nozzle to provide the thrust to propel the aircraft.

![Figure 5.22 In jet engines, the high-temperature and high-pressure gases leaving the turbine are accelerated in a nozzle to provide thrust.](image)

A schematic of a turbojet engine and the $T$-$s$ diagram of the ideal turbojet cycle are shown in Fig. 9–48. The pressure of air rises slightly as it is decelerated in the diffuser. Air is compressed by the compressor. It is mixed with fuel in the combustion chamber, where the mixture is burned at constant pressure. The high-pressure and high-temperature combustion gases partially expand in the turbine, producing enough power to drive the compressor and other equipment. Finally, the gases expand in a nozzle to the ambient pressure and leave the engine at a high velocity.

In the ideal case, the turbine work is assumed to equal the compressor work. Also, the processes in the diffuser, the compressor, the turbine, and the nozzle are assumed to be isentropic. In the analysis of actual cycles, however, the irreversibilities associated with these devices should be considered. The effect of the irreversibilities is to reduce the thrust that can be obtained from a turbojet engine.
The thrust developed in a turbojet engine is the unbalanced force that is caused by the difference in the momentum of the low-velocity air entering the engine and the high-velocity exhaust gases leaving the engine, and it is determined from Newton’s second law. The pressures at the inlet and the exit of a turbojet engine are identical (the ambient pressure); thus, the net thrust developed by the engine is

\[ F = (mV_{\text{exit}} - mV_{\text{inlet}}) = m(V_{\text{exit}} - V_{\text{inlet}}) \quad (N) \]

where \( V_{\text{exit}} \) is the exit velocity of the exhaust gases and \( V_{\text{inlet}} \) is the inlet velocity of the air, both relative to the aircraft. Thus, for an aircraft cruising in still air, \( V_{\text{inlet}} \) is the aircraft velocity.

The power developed from the thrust of the engine is called the propulsive power \( \dot{W}_p \), which is the propulsive force (thrust) times the distance this force acts on the aircraft per unit time, that is, the thrust times the aircraft velocity

\[ \dot{W}_p = FV_{\text{aircraft}} = m(V_{\text{exit}} - V_{\text{inlet}})V_{\text{aircraft}} \quad (kW) \]